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Media of Exchange in Economies with Differential Information

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Abstract

We study an exchange economy where agents have differential information about the quality of goods. Differential information generates specific patterns of net-trades where media of exchange are essential. We provide a rigorous definition of a medium of exchange. We prove the following results: (1) Media of exchange do not exist in economies with symmetrically informed agents. (2) Media of exchange improve the efficiency of the economy. (3) In the presence of some form of auditing, physical goods can be replaced by tokens as media of exchange. The reverse need not hold.

Keywords: Monetary Theory, Media of Exchange, Differential Information Economies, Optimal Contracts.

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1 Introduction

Some of the oldest and most fundamental questions in monetary theory concern the use of media of exchange. Among these questions are the following:

(1) What is a medium of exchange?

(2) Under what circumstances do media of exchange arise?

(3) Do media of exchange improve the efficiency of the economy?

(4) Can an intrinsically worthless good serve as a medium of exchange?

The paper provides a model which answers these and additional questions.

Smith (1776), Jevons (1875), and Wicksell (1911) have investigated the use, the role and the characteristics of media of exchange. They have argued that particular trading patterns arise because trade between agents is subject to frictions. One such friction arises because trade is not centralized. For example, Jevons (1875, pp. 3–4) notes that "the first difficulty of barter is to find two persons whose disposable possessions mutually suit each others wants. There may be many people wanting, and many possessing those things wanted; but to allow of an act of barter there must be a double coincidence, which rarely happens." Jevons concludes that in the absence of a double coincidence of wants, agents will tend to use one particular good on one side of many exchanges. They acquire this good not for consumption purposes but instead to use it in future trades. Such a good is called a medium of exchange.[1]

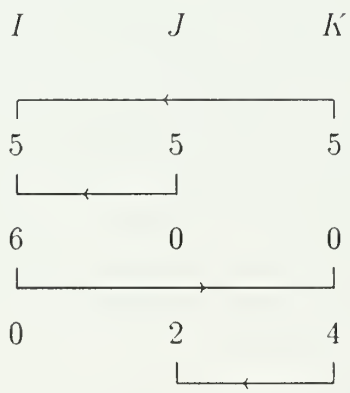
Does the decentralized, pairwise trading described by Jevons cause a friction which leads to a medium of exchange? To answer this question consider the following example by Wicksell, of an economy with three agents $i = 1, 2, 3$, and three goods $j = 1, 2, 3$. Each agent i possesses good $j = i$. However, agent 1 only likes good 2, agent 2 only likes good 3, and agent 3 only likes good 1. Thus, there is an absence of double coincidence of wants in pairwise trading. Nevertheless, in the absence of any additional friction, the agents can agree to execute the following pairwise trades. Each agent i promises to deliver good i to the agent who likes good i. Trade is conducted with the understanding that agent i will receive the good agent i likes from the person who possesses it. Using such implicit "credit arrangements" trade can be executed without the use of a medium of exchange. Thus, in order to

[1]The ideas contained in the paragraph have a long history. John Chipman informs us that clear statements of these ideas go back at least to John Law (1705).

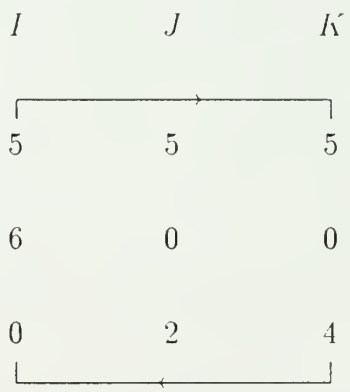
explain why media of exchange are essential for the functioning of the economy a lack of double coincidence of wants is not sufficient and an additional friction is needed.

In our model we consider an economy where trade between agents is pairwise. However, as an additional friction we propose that goods are of uncertain quality and agents are differentially informed about their quality. We show that differential information leads to patterns of trade in which goods are acquired not for consumption purposes but rather for retrading. This pattern of trade is an essential feature of the economy. In particular, it is necessary for the implementation of Pareto efficient and individually rational allocations. In other words, media of exchange arise not because of a lack of double coincidence of wants, but as a result of a lack of double coincidence of information.

In order to provide intuition for why media of exchange arise in our model, we now give a verbal description of a situation with a lack of double coincidence of information. Consider an economy with three agents $i = I, J, K$ and three goods. Think of good 1 as cigarettes, and goods 2 and 3 as Romanian and French wine, respectively. Cigarettes and French wine are of known quality, whereas Romanian wine can be either good or bad quality. The quality is known to agents I and K, but it is unknown to agent J. Moreover, agent J does not like Romanian wine. The endowments are given by $e^I = (5, 6, 0)$, $e^J = (5, 0, 2)$ and $e^K = (5, 0, 4)$. Then a pattern of Pareto efficient trades (derived in Section 4.3) involving no indirect trades (no retrading) is given by



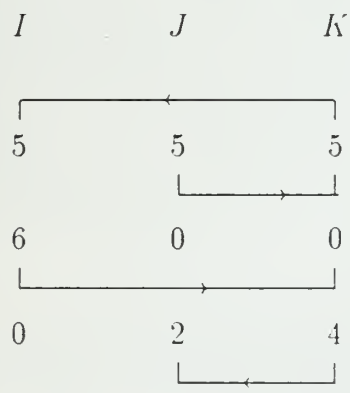
Trades in the good state



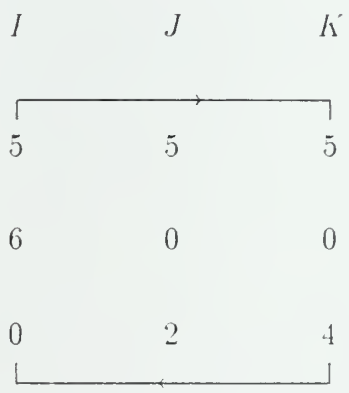
Trades in the bad state

Thus, in a Pareto efficient allocation, agent K will receive some Romanian wine from agent I, and give some French wine to agent J in the good state. In the bad state agent J does not trade. Agent I will then have the incentive to tell agent J the following: "I gave agent K good quality Romanian wine. You must therefore give me some cigarettes. In compensation you will receive French wine from agent K." Clearly, agent I has the incentive to make such announcement regardless of the quality of the wine. Similarly, agent K always has the incentive to tell agent J that the Romanian wine is of bad quality and consequently agent K need not deliver any French wine to agent J. Thus, in contrast to the case of symmetric information, differential information may require some agents to make trades which are contingent on some information they cannot observe. This induces an incentive problem which makes implicit credit arrangements without media of exchange problematic.

How does a medium of exchange resolve this incentive problem? Consider the following pattern of trade.



Trades in the good state



Trades in the bad state

There is no trade between agents I and J. However, agent J now transfers cigarettes to agent K who in turn delivers it to agent I if the good state is announced. In the bad state trade occurs as before. Now agent I does not gain from misreporting the state to agent J because they never trade. Moreover, agent K does not gain from misreporting the state to agent J. because he/she is compensated in cigarettes for giving up French wine in

the good state. Thus, quid pro quo trade—i.e., goods are directly traded for other goods and not traded on credit—arises in this example endogenously to overcome incentive problems and the use of cigarettes as medium of exchange improves the efficiency of the economy.

There is a considerable literature that models media of exchange. This literature includes work by Aiyagari and Wallace (1992), Feldman (1973), Freeman (1989), Harris (1979), Jones (1976), King and Plosser (1986), Kiyotaki and Wright (1989), Oh (1989), Ostroy and Starr (1974), Townsend (1980), Townsend (1989), Williamson and Wright (1991). We now survey a number of these papers that add additional frictions to the pairwise trading restriction. For example, Ostroy and Starr (1974) and (1990) describe one-period models in which agents trade pairwise at competitively determined prices with the requirement that trade is quid pro quo.[2] Townsend (1980) considers an intertemporal model in which spatial separation leads to the use of media exchange. Kiyotaki and Wright (1989) describe an intertemporal framework in which agents are randomly matched and goods have different storage costs. A surprising result of their model is that media of exchange need not be goods with the least storage costs. Williamson and Wright (1991) add private information about the quality of goods to the Kiyotaki and Wright (1989) model. Our model is closest to the tradition surveyed by Ostroy and Starr (1990). However, our model differs in a number of ways. First, it does not assume that trade occurs quid pro quo, rather quid pro quo trade can arise endogenously. Second, it models differential information. Finally, our model differs from the papers cited above, by employing an optimal contracting framework as in Townsend (1987) and Townsend (1989).

Our paper is organized as follows. We first describe the differential information economy. Second, we state a rigorous definition of medium of exchange based on the idea that a medium of exchange is an object with the following properties: It is acquired not solely to be consumed but to be exchanged for some other object and the economy cannot function equally without it. Third, we provide conditions under which no media of exchange exist. Fourth, we provide an example of a differential information economy with a medium of exchange. Fifth, we distinguish between intermediated sales of goods and the use of goods as media of exchange. Finally, we pro-

[2]In their model this means that the value of all net-trades at competitive prices must be zero.

vide conditions under which an intrinsically worthless good (token) can serve as a medium of exchange.

2 The Model

Consider an economy with I agents, denoted by $i \in I$. There are n consumption goods. Each good is available in one of many possible quality grades. In particular, the quality of good k is determined by a number $q \in Q$. In most cases Q can be assumed to be a subset of $\mathbb{R}_+$. Thus, a commodity can be specified by the underlying physical good k, $k = 1, \ldots, n$, and its quality q.

There is uncertainty about the quality of the consumption goods. This is modeled as follows. Let $(\Omega, \mathcal{A}, P)$ be a probability space. For $k = 1, \ldots, n$ let $q_k : \Omega \to Q$ be random variables, such that $q_k(\omega)$ is the quality of good k in state ω. A consumption plan of agent i is given by a mapping $x^i : \Omega \to \mathbb{R}^n$. Each agent i has an initial endowment given by $e^i : \Omega \to \mathbb{R}^n$.

The information of agent i is given by a measurable partition[3] $\mathcal{F}_i$ of Ω. At the time of trade agent i knows the event of the partition which contains the true state of the economy ω.[4] Without loss of generality we can assume that the initial endowment e^i of agent i is measurable with respect to his/her information $\mathcal{F}_i$. We assume that the preferences of each agent $i = 1, \ldots, I$ in state $\bar{\omega}$ given the information $\mathcal{F}_i$ can be represented by a von Neumann Morgenstern utility function

$$E\left[u^i\left(x_1(\omega), \ldots, x_n(\omega), q_1(\omega), \ldots, q_n(\omega)\right) \,\middle|\, \mathcal{F}_i\right](\bar{\omega}).$$

[3] If Ω is finite, then a measurable partition is simply a collection of sets A_j, $j \in J$ such that the union of all sets equals Ω, i.e., $\bigcup_{j \in J} A_j = \Omega$, and such that all sets are pairwise disjoint, i.e., $A_j \cap A_k = \emptyset$, for all $j \neq k$.

In general, a measurable partition of Ω is a collection of sets A_j, $j \in J$, with the following properties: (a) J is finite or countable; (b) $A_j \in \mathcal{A}$ for every j, i.e., the sets are measurable; (c) $\bigcup_{j \in J} A_j = \Omega$; (d) $A_j \cap A_k = \emptyset$ for all $j \neq k$.

[4] For example, let $\Omega = \{a, b, c, d\}$, and assume the information of a particular agent is given by the partition $\mathcal{F} = \{\{a\}, \{b, c, d\}\}$. If a is the true state of nature, then the element of $\mathcal{F}$ which contains a is $\{a\}$. In other words the knows exactly the true state of nature. On the other hand, assume that b is the true state of nature. Then $\{b, c, d\}$ is the element of $\mathcal{F}$ containing b. Thus, the agent only knows that either b, c, or d is the true state of nature.

The economy described above is similar to the differential information economies described in Radner (1968), Wilson (1978), and Yannelis (1991). However, the key difference is that trading in our economy must be executed pairwise. Thus, the trade of agent i is described by the net-trades $z^{i,j}$ with all other agents j. A collection of net-trades for all agents is feasible if and only if $z^{i,j} = -z^{j,i}$ for every $i, j \in I$. Thus, $z^{i,i} = 0$ for all i. We assume that a trade $z^{i,j}$ can only be observed by agents i and j. As a consequence, a trade between two agents does not reveal any information to other agents and information revelation therefore occurs pairwise. Although we consider a one period economy, for better intuition we can think of the following timing of events. First agents sign contracts. In the tradition of Townsend (1987) we can think of a social planner specifying Pareto efficient and individually rational contracts. The planner knows the structure of the economy but has no information on the state of nature. At the time when contracts are signed agents have only their own information. Agents will agree to a contract if it is individually rational. Once the contract is signed agents independently announce their information pairwise and all trades specified by the contract are executed.[5] Finally, agents consume. At the time of consumption all agents need not yet be fully informed about the quality of the goods they own and of those which they have received in the trading period.

3 Optimal Trading Contracts

In the presence of differential information arbitrary net-trades are not implementable since agents have an incentive to misreport their information. Moreover, in our economy agents trade pairwise and information revealed in each of the pairwise trades is not revealed to any other agent. As a consequence, it is possible for any agent to make different reports to different trading partners. In the following definition we introduce an incentive constraint which ensures that each agent i reveals, his/her information truthfully

[5] We assume that agents cannot renege on their contracts. Thus, there is ex-post enforcement by the social planner. It is important to note that if contracts are incentive compatible as described in the next section, the social planner need not have any information in order to enforce the contracts.

in all of his/her trades.[6]

Definition 1. *Let $z^{i,j}$, $j \in I$ be a collection of net-trades for agent i. Let e^i be the initial endowment of agent i. Then the net-trades are Bayesian incentive compatible if and only if*
(i) $z^{i,j}$ is $\mathcal{F}_i \vee \mathcal{F}_j$-measurable[7] for all $j \in I$;
(ii)

$$E\left[u\left(e^i(\omega) + \sum_{j\in I} z^{i,j}(\omega), q(\omega)\right) \mid \mathcal{F}_i\right](\bar{\omega}) \geq E\left[u\left(e^i(\omega) + \sum_{j\in I} z^{i,j}(\bar{\omega}^j), q(\omega)\right) \mid \mathcal{F}_i\right](\bar{\omega}), \qquad (1)$$

for every $\bar{\omega} \in \Omega$, and for all $\bar{\omega}^j \in F_j(\bar{\omega})$, where $F_j(\bar{\omega})$ is the element of the partition $\mathcal{F}_j$ which contains $\bar{\omega}$.[8]

Condition (i) means that the two agents can execute the trade $z^{i,j}$ with the information available to the two agents. Condition (ii) specifies that truthful revelation of information is optimal. The left-hand side of inequality (1) is the expected utility of agent i from truthful reporting given the agent's information. The right-hand side of (1) is the expected payoff if agent i reports $\bar{\omega}^j$ in his/her trade with agent j. As a consequence, the net-trade $z^{i,j}(\bar{\omega}^j)$ is executed. The condition $\bar{\omega}^j \in F_j(\bar{\omega})$ means that agent j is not able to detect the misreport.[9]

[6]The incentive constraint introduced in Definition 1 is an adaptation of the Bayesian incentive constraint in Myerson (1984, p. 74).

[7]$\mathcal{F}_i \vee \mathcal{F}_j$ is the partition generated by the partitions $\mathcal{F}_i$ and $\mathcal{F}_j$ (intuitively, it is the union of the two partitions). $\mathcal{F}_i \vee \mathcal{F}_j$-measurability of $z^{i,j}$ means that $z^{i,j}$ is a constant function on each element of the partition $\mathcal{F}_i \vee \mathcal{F}_j$.

[8]Formally, $F_j(\omega) = \bigcap_{F\in\mathcal{F}_j,\, F\ni\omega} F$.

[9]We do not allow misreports in trades between agents i and j which one of the agents can immediately detect. For example, assume there are two states a and b, which both agents i and j can observe. Assume the true state is a. Then it is by assumption not possible, say for agent i, to report that the true state is b. What mechanism can exclude such misreports? For example, assume that information can be revealed in court although at a high cost of "effort." Then the court can penalize agent i and compensate agent j. Consequently, i has no incentive not to go to court and agent j will therefore report truthfully. Moreover, the court is not needed in an optimal contract if court and effort costs

In the following we characterize optimal contracts between agents. These are trades between the agents which are Bayesian incentive compatible for each agent, individually rational and Pareto efficient.[10]

Definition 2. *An optimal contract between the agents consists of a collection of net-trades $z^{i,j}$ with the following properties:*
(i) (feasibility) $z^{i,j} = -z^{j,i}$ for all $i, j \in I$.
(ii) (individual rationality)

$$E\left[u\left(e^i + \sum_{j \in I} z^{i,j}, q\right) \mid \mathcal{F}_i\right](\bar{\omega}) \geq E\left[u\left(e^i, q\right) \mid \mathcal{F}_i\right](\bar{\omega}),$$

for each consumer i and for every state $\bar{\omega} \in \Omega$.
(iii) (Incentive compatibility) The trades $z^{i,j}$, $j \in I$ are Bayesian incentive compatible for every consumer i.
(iv) (Pareto efficiency) There does not exist a collection of net-trades $\tilde{z}^{i,j}$ which fulfill conditions (i), (ii), and (iii) and such that

$$E\left[u\left(e^i + \sum_{j \in I} \tilde{z}^{i,j}, q\right) \mid \mathcal{F}_i\right](\bar{\omega}) \geq E\left[u\left(e^i + \sum_{j \in I} z^{i,j}, q\right) \mid \mathcal{F}_i\right](\bar{\omega}),$$

for all agents $i = 1, \ldots, I$ and for all $\bar{\omega} \in \Omega$, with the strict inequality holding for at least one agent i on a set of positive measure.

We now continue with an example that illustrates how to determine the structure of optimal contracts. The detailed computations involve only elementary algebra and can be found in the Appendix.

Example 1. Assume there are two agents denoted by I and J. There are three consumption goods, however, there is product uncertainty only about good number two. The initial endowments of agents I and J are given by $e^I = (5, 4, 0)$ and $e^J = (5, 0, 2)$, respectively. Let $\Omega = \{g, b\}$ and $P(\{g\}) = P(\{b\}) = 0.5$. Let $q_2(g) = 1$ and $q_2(b) = 0$. Thus, q_2 assumes the

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are sufficiently high. We therefore abstract from misreports which can be detected and do not include a verification mechanism in the Pareto problem for simplicity of exposition.

[10]The notions of Pareto efficiency and individual rationality are similar to those in Yannelis (1991, Definitions 3.1.2 and 3.1.3).

values 0 and 1 with probability 0.5 each. The utility function of both agents is given by $u(x_1, x_2, x_3, q_1, q_2, q_3) = x_1 + \sqrt{q_2 x_2 + x_3}$. The information sets of the agents are given by $\mathcal{F}_I = \{\{g\}, \{b\}\}$, and $\mathcal{F}_J = \{\{g, b\}\}$. In this example, optimal contracts have the following structure: No-trade of goods two and three occurs in the good state (g). In the bad state (b), agent J trades one unit of good three to agent I.

There are also examples in which no trade occurs. In particular, choose $e_3^I = 2$ for the initial endowment of agent I instead of $e_3^I = 0$ leaving all other parameters from Example 1 unchanged. Then we can use similar arguments as in Example 1 to show that no-trade of goods two and three will occur in the good state. However, since the marginal rates of substitution of both agents in the bad state are the same, there are no gains from trading in the bad state. Hence agents will not trade and consume their initial endowment.

4 Media of Exchange

4.1 Definition of Media of Exchange

There are various definitions of what constitutes a medium of exchange. We list briefly the most important definitions. In particular, a medium of exchange is

(1) an object with a high transaction velocity;
(2) an object which is on one side of many exchanges;
(3) an object generally acceptable in trade;
(4) an object which is taken in exchange only to be exchanged for some other object;

Definition 4 can be found in Jevons (1875) and Wicksell (1911). According to Wicksell (1911, p. 15) a medium of exchange is "an object which is taken in exchange, not for its own account, i.e. not to be consumed by the receiver or to be employed in technical production, but to be exchanged for something else within a longer or shorter period of time." Our definition is based on this insight. We now formalize in our model what it means for a good to be exchanged only for the purpose of retrading it. In particular, let $z^{i,J}$ be a collection of net-trades for all agents. Then for every agent i and for every good k we define a notion of a trade velocity.

Definition 3. *Agent i's trade velocity of good k in state ω given the net-trades $z^{i,J}$ is defined by*

$$v_k^i\left(z^{i,J}\right)(\omega) = \sum_{j\in I}\left|z_k^{i,j}(\omega)\right| \Big/ \left|\sum_{j\in I} z_k^{i,j}(\omega)\right|, \tag{2}$$

if $\sum_{j\in I}\left|z_k^{i,j}(\omega)\right| = 0$ then we set $v_k^i\left(z^{i,J}\right)(\omega) = 1$, otherwise, if the denominator is zero and the numerator is nonzero we define $v_k^i\left(z^{i,J}\right)(\omega) = \infty$.

In our model all goods are consumption goods. However, it is possible that a particular good is traded in excess of its consumption need. In such a case, the trade velocity (velocity as in Definition 3) is strictly greater than one. If a good is only traded by an agent for purposes other than consumption (i.e, $\sum_{j\in I} z_k^{i,j}(\omega) = 0$) then the velocity of this good is infinite. Thus, our definition of a medium of exchange will require that the velocity is strictly greater than one. However, our definition requires an additional property which is not contained in the definition of Wicksell. Consider the following example:

Example 2. There are three agents I, J, K in an economy with no uncertainty. There is only one consumption good. Consider the following pairwise trades: $z^{I,J} = z^{J,K} = z^{K,I} = 1$. Clearly, feasibility requires $z^{J,I} = z^{K,J} = z^{I,K} = -1$. Obviously, all net-trades cancel each other out at the individual and aggregate level, and thus they implement the only optimal allocation in this economy (i.e, Pareto efficient and individually rational allocation). The velocity, however, for each agent is infinite.

Strictly speaking the good in Example 2 classifies as a medium of exchange according to Wicksell's definition. However, the economy can clearly function equally without good one being traded. If the good is not traded it has a velocity of one and does not qualify any more as a medium of exchange. A similar argument as in Example 2 can be made for all goods in any three or more agent economy. As a consequence, if the velocity of a good is our only element of a definition of medium of exchange, all goods will automatically qualify as a medium of exchange. A definition of medium of exchange must therefore be based on an additional requirement. In particular, if a good is a *medium of exchange* then it is an *object which is acquired not solely to be consumed but to be exchanged for some other object;* and *the economy*

cannot function equally without it. In our model, this means that retrading is essential to implement a particular optimal allocation.

In Definition 4 we formalize what we mean by the statement that retrading is essential in the economy. In particular, it should not be possible to replace the original trades by alternative trades which implement the same allocation and which do not require the good to be a medium of exchange. We now give our definition.

Definition 4. *Let $z^{i,j}$ be an optimal contract in a differential information economy. Then good k is a* **medium of exchange** *if and only if the following two conditions hold:*

(i) There exists an agent $\ell \in I$ such that $v_k^f(z^{i,j})(\omega) > 1$ on a set of positive measure;

(ii) There does not exist an optimal contract $\tilde{z}^{i,j}$ with

(a) $\sum_{j\in I} \tilde{z}^{i,j} = \sum_{j\in I} z^{i,j}$ for all agents $i \in I$;

(b) $\tilde{z}_m^{i,j} = z_m^{i,j}$ for all goods m for which there exists an agent i such that $v_m^i(\tilde{z}^{i,j}) > 1$.

(c) $v_k^i(\tilde{z}^{i,j})(\omega) = 1$ a.e. for all agents $i \in I$.

Condition (i) requires that a medium of exchange has a velocity greater than one. Condition (ii) formalizes the idea that the economy cannot function equally without the medium of exchange. In particular, condition (a) requires that the alternative trades result in the same allocation. (b) is important in cases with more than one medium of exchange. For example, assume that there are two goods which are perfect substitutes which are both traded with a velocity greater than one. Then it is always possible to make all trades with velocity greater than one in only one of the goods. Consequently without (b) no good which has a sufficiently good substitute will classify as a medium of exchange. (c) states that the velocity of good k is one for all agents.[11]

4.2 Some Elementary Results

In this section we collect some elementary results which follow immediately from the definition.

[11] Definition 4 can easily be extended to more general spaces of contracts. For example, we need not assume that contracts are Pareto efficient. All of the results still go through under this more general specification.

Fact 1. No medium of exchange exists if the endowment is already an optimal allocation.

Fact 2. No medium of exchange exists in any economy with less than three agents.

Fact 3. No medium of exchange exists in economies with symmetrically informed agents.

Proof. If the endowment is already Pareto efficient, then the endowment itself is an optimal allocation. Thus, the optimal allocation can be implemented by the trivial trades $\tilde{z}_k^{i,j} = 0$ for all goods k. Thus, $v_k^{\ell}(\tilde{z}^{i,j}) = 1$ for all goods. Thus, condition (ii) of Definition 4 implies that no good is a medium of exchange. This proves Fact 1.

Fact 2 follows immediately from the fact that there is at most one net-trade per agent in these economies. Hence the velocity of all goods must be one.

We now show Fact 3. Let $z^{i,j}$ be an arbitrary collection of net-trades. Then we can define for any good k new net-trades $\tilde{z}^{i,j}$ with velocity one by using the following construction:

Let $z_k^{i,j}$ denote the net-trades in good k. Then define the new net-trades $\tilde{z}_k^{i,j}$ as follows: Let $P = \{i \in I \mid \sum_{j \in I} z_k^{i,j} > 0\}$, and let $N = \{i \in I \mid \sum_{j \in I} z_k^{i,j} < 0\}$. For all agents $i \notin P \cup N$ and for all $j \in I$ define $\tilde{z}_k^{i,j} = 0$. For every $i \in P$ now define $\alpha_i = \sum_{j \in I} z_k^{i,j} \,/\, \sum_{i \in P} \sum_{j \in I} z_k^{i,j}$. Similarly, for every $i \in N$ define $\alpha_i = \sum_{j \in I} z_k^{i,j} \,/\, \sum_{i \in N} \sum_{j \in I} z_k^{i,j}$. For all agents i we now define the alternative trades

$$\tilde{z}_k^{i,j} = \begin{cases} -\alpha_i \sum_{r \in I} z_k^{j,r} & \text{if } (i,j) \in N \times P \text{ or } (i,j) \in P \times N; \\ \tilde{z}_k^{i,j} = 0 & \text{otherwise.} \end{cases}$$

By construction the net-trades are feasible and $\sum_{j \in I} \tilde{z}_k^{i,j} = \sum_{j \in I} z_k^{i,j}$ for all agents $i \in I$. We now check that the velocity of good k is 1 for all agents.

The assertion follows immediately for agents $i \notin P \cup N$. We now show it for agents $i \in P$. Note that by construction $\tilde{z}_k^{i,j} \geq 0$ for all $i \in P$ and for all $j \in I$. Thus, the numerator and the denominator in the definition of velocity are the same. The argument for agents $i \in N$ is similar.

Finally, it remains to prove that $\tilde{z}^{i,j}$ is Bayesian incentive compatible and $\mathcal{F}_i \vee \mathcal{F}_j$-measurable. Since agents' information is symmetric, $z^{i,j}$ is $\mathcal{F}_i$-measurable for all $i \in I$. Since $\tilde{z}^{i,j}$ is a composition of functions $z^{i,j}$ it is $\mathcal{F}_i$,

and hence $\mathcal{F}_i \vee \mathcal{F}_j$-measurable. Finally, because of symmetric information $\mathcal{F}_i = \mathcal{F}_j$ and hence every agent can verify all of his/her net-trades. Misreports are consequently not possible.

The facts above list cases in which our intuition tells us that media of exchange serve no purpose. These cases consider extreme preference patterns, endowment patterns, differential information patterns, and numbers of agents. In each case our definition accords with intuition.

We now discuss Facts 1 to 3 and indicate how they fit into existing thought in monetary economics.

Remark 1. Fact 1 reflects the broader truth that selfish agents do not trade if endowments are Pareto efficient. It also reflects the need for frictions in monetary economics as discussed in Remark 2 below. Similarly, Fact 2 verifies an additional case where a medium of exchange should not exist.

Remark 2. Fact 3 provides a rigorous proof of a Folk Theorem in monetary economics. The Folk Theorem is that some friction is needed to explain media of exchange. In our case trading frictions arise as a consequence of differential information. It is also important to note that full information is a special case of symmetric information. Thus, Fact 3 indicates that no medium of exchange exists in a full information economy. As a consequence, "Wicksell triangles" [cf., Wicksell (1911, pp. 16ff) or Niehans (1978, p. 101)] do not require media of exchange in the absence of an additional friction. In order to illustrate this point consider the following simple example by Wicksell (1911, pp. 16ff).

Assume that there are three countries $i = 1, 2, 3$. There are three goods. Each country i is endowed with one unit of good i. Country i only likes good $i + 1$ mod(3).[12] Thus, in the Pareto efficient allocation each country i consumes one unit of good $i + 1$ mod(3). In Wicksell (1911) this allocation is implemented by having one country serve as an intermediary. For example, choose county 1 as the intermediary. Then the net-trades between the countries are given by $z^{1,2} = (0, 1, -1)$, $z^{1,3} = (-1, 0, 1)$ and $z^{2,3} = (0, 0, 0)$.[13] Clearly, good 3 has a velocity greater than one. Alternatively, however, the

[12]That is, country 1 likes good 2, country 2 likes good 3, and county 3 likes good 1.

[13]The remaining trades follow from feasibility. In particular, $z^{2,1} = (0, -1, 1)$, $z^{3,1} = (1, 0, -1)$ and $z^{3,2} = (0, 0, 0)$.

trades $z^{1,2} = (0, 1, 0)$, $z^{1,3} = (-1, 0, 0)$ and $z^{2,3} = (0, 0, 1)$ also implement the allocation. Thus, according to definition 4 no good is a medium of exchange.

In his model, Wicksell (1911) assumes that trade must be pairwise *and* quid pro quo, i.e., goods can only be swapped for each other. In our model, however, agents can receive goods "on credit." For example, agent 1 gives his endowment to agent 3 without immediate compensation, but with the understanding that he/she will receive one unit of good 2 from agent 2. In other words, although trade is executed pairwise it is agreed upon by all agents. In the case of full information the pairwise trading assumption does not impose any additional restriction in our framework. However, in the case of differential information the pairwise trading restriction implies that information is only revealed pairwise. This information revelation mechanism differs substantially from mechanisms where information revelation is centralized.

4.3 An Economy with a Medium of Exchange

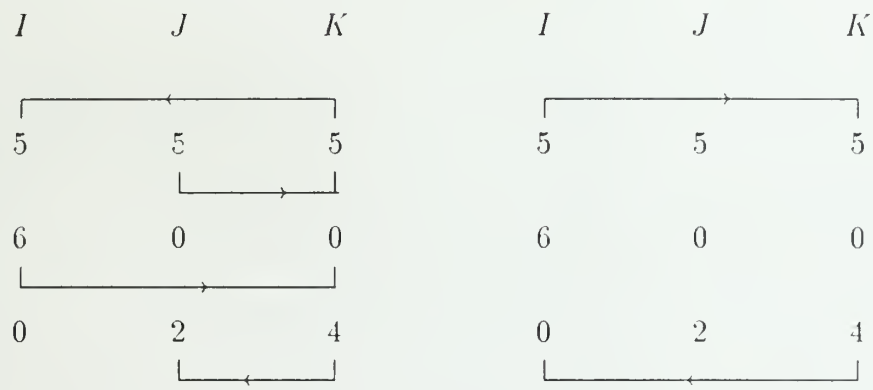
In section 4.2 we established some general results on non existence of media of exchange. In this section we provide a simple example of a differential information economy with a medium of exchange, thus showing that Definition 4 is not vacuous. In particular, we prove the existence of a good which is a medium of exchange for one of the consumers.

The example illustrates an important proposition in monetary economics. The proposition is that the use of a medium of exchange increases the efficiency of the economy. In our framework this proposition can be rephrased as follows: There is an optimal allocation that cannot be achieved without a medium of exchange. To prove that media of exchange are necessary to achieve all optimal allocations, it is clear that the economies described in Section 4.2. must be ruled out.

The main idea of the example is as follows: We consider an economy with three agents I, J, K with a lack of double coincidence of information between some agents. In such a situation trade will tend to occur in goods about which agents share common information. This leads to a particular pattern of trade in which a good which is recognizable by all agents functions as a medium of exchange. In particular, agent J has no information about good two. Good two can be either of good or bad quality. Since in the absence of informational problems goods two and three are perfect substitutes, K

acquires more of good two from agent I, in order to trade some of good three to agent J if good two is of good quality. If the quality of good two is bad, then there is no trade between agents I and J. In order to ensure that K announces the state truthfully in the trade with agent J, good one must be used as medium of exchange.

Example 3.[14]



Trades in state $\omega = g$ *Trades in state $\omega = b$*

Assume that there are three agents denoted by I, J, K. There are three goods. Endowments are given by $e^I = (5,6,0)$, $e^J = (5,0,2)$, and $e^K = (5,0,4)$. There is uncertainty about the quality of good two but no uncertainty about goods one and three. Then $q_1 = q_3 \equiv 1$. We now describe q_2. Let $\Omega = \{g,b\}$ and let $P(\{g\}) = P(\{b\}) = 0.5$. We assume that $q_2(g) = 1$ and $q_2(b) = 0$. All agents have the same utility function given by $u(x_1,x_2,x_3,q_1,q_2,q_3) = x_1 + \sqrt{q_2 x_2 + x_3}$. The information of the agents is given by $\mathcal{F}_I = \mathcal{F}_K = \{\{g\},\{b\}\}$, and $\mathcal{F}_J = \{\{g,b\}\}$. In other words, agents I and K have full information whereas agent J has no information about the quality of good two.

We now show that the following collection of net-trades describe an optimal contract: $z^{I,J}(\omega) = (0,0,0)$ for $\omega = g,b$; $z^{I,K}(g) = (\sqrt{6}-2,-2,0)$,

[14]This example corresponds to the example discussed in the introduction. However, we assume that agent J also likes good 2 in order to show that the result does not depend upon a very specific choice of utility function. It should be clear to the reader that the same result holds if agent J does not derive utility from good 2.

$z^{I,K}(b) = (-1, 0, 2)$, and $z^{J,K}(g) = (\sqrt{2} - 2, 0, 2)$, $z^{J,K}(b) = (0, 0, 0)$. First, verify that the contract is individually rational for each agent. The individual rationality constraints for agent I are

$$5 + z_1^{I,K}(g) + \sqrt{6 + z_2^{I,K}(g)} \geq 5 + \sqrt{6};$$
$$5 + z_1^{I,K}(b) + \sqrt{z_3^{I,K}(b)} \geq 5.$$

For agent J the Bayesian individual rationality constraint is

$$1/2\left(5 + z_1^{J,K}(g) + \sqrt{2 + z_3^{J,K}(g)}\right) + 1/2\left(5 + \sqrt{2}\right) \geq 5 + \sqrt{2}.$$

Finally, the individual rationality constraints for agent K are

$$5 - z_1^{I,K}(g) - z_1^{J,K}(g) + \sqrt{4 - z_2^{I,K}(g) - z_3^{J,K}(g)} \geq 5 + \sqrt{4};$$
$$5 - z_1^{I,K}(b) + \sqrt{4 - z_3^{I,K}(b)} \geq 5 + \sqrt{4}.$$

Second, we must verify that the contract is incentive compatible. Because there is no trade between agent I and agent J, we only need to check the incentive constraint for agent K. Moreover, note that agent K cannot misreport to agent I because they both have symmetric information. These constraints are given by

$$5 - z_1^{I,K}(g) - z_1^{J,K}(g) + \sqrt{4 - z_2^{I,K}(g) - z_3^{J,K}(g)}$$
$$\geq 5 - z_1^{I,K}(g) + \sqrt{4 - z_2^{I,K}(g)};$$
$$5 - z_1^{I,K}(b) + \sqrt{4 - z_3^{I,K}(b)}$$
$$\geq 5 - z_1^{I,K}(b) - z_1^{J,K}(g) + \sqrt{4 - z_3^{I,K}(b) - z_3^{J,K}(g)}.$$

It is straightforward to check that the constraints hold for the contract specified above. Finally, we check that the contract is Pareto efficient. First, note that the final allocation in our example is Pareto efficient under complete information. If the contracts are not Pareto efficient then there must exist a state $\omega \in \Omega$ and an alternative contract such that all agents can be made better off in state ω. However this violates the fact that the final allocation

derived from the original contract is Pareto efficient under full information.[15]

Since we have established that $z^{i,j}$ is an optimal contract we can analyze whether one (or more) of the three goods is a medium of exchange. Clearly, the velocities of goods two and three is one for all agents and for all states. However, note that the velocity of good one of agent K in state $\omega = g$ is given by $v^K(z^{i,j}))(g) = (\sqrt{6} - \sqrt{2}) \,/\, (4 - \sqrt{2} - \sqrt{6})$, and hence $v^K(z^{i,j}))(g) > 1$. In order to establish that good one is a medium of exchange for agent K it remains to prove that we cannot find an optimal contract $\tilde{z}^{i,j}$ which implements the same allocation and for which $v_1^k(\tilde{z}^{i,j})(\omega) = 1$ for all ω and for all agents k. Note that any trade with velocity one for agent K implies trade between agents I and J. Denote this trade by $\tilde{z}_1^{I,J}$. Moreover, we must have $\tilde{z}_1^{I,J}(g) \neq \tilde{z}_1^{I,J}(b)$. Furthermore, note that by definition the trade in all other goods must remain the same. Hence, $\tilde{z}^{I,J}(g) = (z_1^{J,K}(g), 0, 0)$ and $\tilde{z}^{I,J}(b) = (0, 0, 0)$. However, such trades are not Bayesian incentive compatible. In particular, agent I will always announce the state $\bar{\omega}$ with $\bar{\omega} = \arg\max_{\omega \in \Omega} \tilde{z}_1^{I,J}(\omega)$. Thus, it is essential to have $v^K(z^{i,j}))(g) > 1$ in the optimal contract. Consequently good one is a medium of exchange.

In the above example agent K is an intermediary in trade between agents I and J. In particular, there is no trade between agents I and J and agent

[15]Formally, Pareto efficiency can be derived as follows. We evaluate whether there are Pareto improvements from the conjectured optimal contract. Therefore, consider the following optimization problem

$$\begin{aligned} \max_{x_1, x_2, y_1, y_2} \; & 1/2\sqrt{2 + x_1} + 1/2\sqrt{4 + x_2} + 1/2y_1 + 1/2y_2, \\ \text{subject to} \; & (1) \; \sqrt{2 - x_1/2} - y_1/2 = \sqrt{2}, \\ & (2) \; \sqrt{4 - x_2/2} - y_2/2 = 2. \end{aligned}$$

This problems maximizes the expected utility of agent J subject only to the individual rationality constraints of agents I and K in the bad state (1) and in the good state (2). Without loss of generality, we treat agents I and K symmetrically. After inserting (1) and (2) into the objective, the first order conditions are

$$\frac{1}{2\sqrt{2 + x_1}} = \frac{1}{2\sqrt{2 - x_1/2}} \quad \text{and} \quad \frac{1}{2\sqrt{4 + x_2}} = \frac{1}{2\sqrt{4 - x_2/2}}.$$

One can immediately verify that $x_1 = x_2 = 0$ is the only solution. Thus, the proposed allocation is Pareto efficient in the absence of the incentive constraints. Therefore it is Pareto efficient with the incentive constraints.

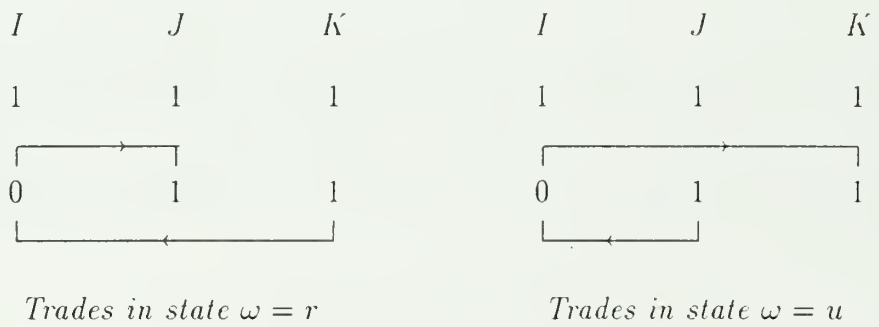
J receives all of his/her goods via agent K. The existence of a medium of exchange for agent K and his/her role as an intermediary are inherently linked in example 3.

4.4 Intermediation versus Media of Exchange

In most economies a substantial fraction of trade runs through intermediaries. For example, goods are often traded on a retail basis rather than purchased wholesale. In our terminology, such goods have a velocity greater than one. In most circumstances, however, these goods would not be considered to be media of exchange. In order to distinguish between intermediated sale of goods and media of exchange, we must refine definition 4. This is the purpose of this section.

How does intermediated trade arise in our model? Consider the following example.

Example 4.



Trades in state $\omega = r$ *Trades in state* $\omega = u$

Assume that there are three agents $i = I, J, K$ and two goods. The endowments are not state-contingent and are given by $e^I = (1, 0)$, $e^J = e^K = (1, 1)$. There is uncertainty about the quality of good 2. For example, think of good 2 as watermelons, which can be either ripe (r) or unripe (u). Thus, let $\Omega = \{r, u\}$ and $q_2(\omega) = \omega$ for $\omega = r, u$. Assume that each state occurs with equal probability. Agent I can determine whether a watermelon is ripe or unripe, i.e., $\mathcal{F}_I = \{\{r\}, \{u\}\}$. On the other hand, agents J and K cannot determine the quality of watermelons, i.e., $\mathcal{F}_J = \mathcal{F}_K = \{\{r, u\}\}$. The utility

functions of the agents I are given by $u^I(x_1, x_2, q) = x_1$ and

$$u^J(x_1, x_2, q) = \begin{cases} x_1 + x_2 & \text{if } q_2 = r; \\ x_1 & \text{otherwise.} \end{cases} \quad u^K(x_1, x_2, q) = \begin{cases} x_1 & \text{if } q_2 = r; \\ x_1 + x_2 & \text{otherwise.} \end{cases}$$

An optimal allocation in this economy is given by $x^I(r) = x^I(u) = (1, 0)$, $x^J(r) = x^K(u) = (1, 2)$, and $x^J(u) = x^K(r) = (1, 0)$.

How can this allocation be implemented? Since agents J and K have no information they cannot achieve this allocation without the help of agent I. Since agent I does not like watermelons (good 2) he/she can intermediate the trade without any incentive problems. Thus, the following trades constitute an optimal contract. In state r, the trades are given by $z^{I,J}(r) = (0, -1)$, $z^{I,K}(r) = (0, 1)$; in state u the opposite occurs. There is no pairwise trade between agents J and K.

Clearly, $v_2^I(z^{i,j}) = \infty$ in both states, however, is good 2 really a medium of exchange in example 4? In fact, good 2 is traded only for its own sake, not to facilitate trade in other goods. Moreover, trade in good 2 does not occur for incentive reasons. In this sense there is intermediated trade in good 2. however, good 2 is not a medium of exchange. We now introduce a refined version of Definition 4 which distinguishes between intermediation and media of exchange. In our refined definition a good is a medium of exchange if it is traded with a velocity in excess of consumption needs and if the economy cannot function equally without it. We now refine what it means for a good to be traded in excess of its consumption needs. In particular, we define trades with minimum velocity.

Definition 5. *Let $z^{i,j}$ be an optimal contract. Then $z^{i,j}$ has minimum velocity in good k if and only if the following does not hold: There exists net-trades $\tilde{z}_k^{i,j}$ in good k with*

(i) $\sum_{j \in I} z_k^{i,j} = \sum_{j \in I} \tilde{z}_k^{i,j}$ *for all agents i;*

(ii) $\tilde{z}_k^{i,j}$ *is $\mathcal{F}_i \vee \mathcal{F}_j$-measurable for all agents i, j;*

(iii) $v_k^i(\tilde{z}_k^{i,j})(\omega) \leq v_k^i(z_k^{i,j})(\omega)$ *for all agents i, where the strict inequality holds on a set of positive measure for at least one agent.*

Conditions (i) and (iii) state that a trade has minimum velocity provided it is not possible to statewise reduce the velocity further without changing the allocation. Condition (ii) reflects the requirement that agents must trade

on their available information. It does not require that trades are incentive compatible.

In our refined definition, a medium of exchange is also a good that facilitates trade in some other goods. In our model this happens because a medium of exchange reduces the incentive problems in exchange. If a good is therefore not traded with more than minimum velocity, it does not qualify as a medium of exchange. This leads us to the following strengthened version of Definition 4. Note that Definition 6 can be derived from Definition 4 simply by replacing the terms "velocity one" by "minimum velocity."

Definition 6. *Let $z^{i,j}$ be an optimal contract in a differential information economy. Then good k is a* **strong medium of exchange** *if and only if the following two conditions hold:*

(i) $z^{i,j}$ does not have minimum velocity in good k.

(ii) There does not exist an optimal contract $\tilde{z}^{i,j}$ with

- *(a) $\sum_{j\in I} \tilde{z}^{i,j} = \sum_{j\in I} z^{i,j}$ for all agents $i \in I$;*
- *(b) $\tilde{z}^{i,j}_m = z^{i,j}_m$ for all goods m for which $\tilde{z}^{i,j}$ does not have minimum velocity.*
- *(c) $\tilde{z}^{i,j}$ has minimum velocity in good k.*

Remark 3. If a good is a strong medium of exchange (according to Definition 6) then it is also a medium of exchange (according to Definition 4). As a consequence Facts 1, 2, and 3 still hold for strong media of exchange.

We now reexamine Examples 3 and 4. In example 3, the minimum velocity in *any* good is 1. This follows for the following reasons. First, trades can be reduced to velocity one trade by the argument used in the proof of Fact 3. Second, the resulting net-trades are $\mathcal{F}_i \vee \mathcal{F}_j$ measurable, since the joint information of each pair of agents corresponds to full information, i.e., $\mathcal{F}_i \vee \mathcal{F}_j = \{\{g\},\{b\}\}$ for all agents $i, j \in I$. Thus, Definition 6 and Definition 4 and hence the two concepts coincide for example 3.

Finally, reconsider example 4. Note that the velocity cannot be reduced any further. In order to reduce the velocity, it would be necessary to have $\tilde{z}^{J,K} \neq 0$. Furthermore, $\tilde{z}^{J,K}$ must be state contingent. This is impossible since both agents have only trivial information. Hence, the original trades have minimum velocity and neither good is a strong medium of exchange.

4.5 Tokens and Credit as Media of Exchange

This section addresses the question whether intrinsically worthless goods (tokens) can serve as media of exchange. Proposition 1 in this section establishes that intrinsically worthless goods in zero-net supply (credit) can serve as media of exchange at least as well or better than goods with an intrinsic value. Finally, we show that a government that issues tokens and commits to collecting the tokens at the end of the period (as in Gale (1978)) can be used as a substitute for the credit mechanism.

We start with a technical Lemma.

Lemma 1. *Let $z_k^{i,j}$ be a collection of $\mathcal{F}_i \vee \mathcal{F}_j$-measurable net-trades in good k. Then there exist $\mathcal{F}_i \vee \mathcal{F}_j$-measurable net-trades $\tilde{z}_k^{i,j}$ with minimum velocity such that $\sum_{j \in I} \tilde{z}_k^{i,j} = \sum_{j \in I} z_k^{i,j}$ for all agents i.*

Proof. See Appendix.

We are now ready to prove the main result of this section.

Proposition 1. *Assume that the utility functions of all agents are monotone. Furthermore, assume that there exists a good k which is in zero net-supply and from which agents do not derive utility. If a differential information economy has a strong medium of exchange, then good k is also a strong medium of exchange.*

Proof. Without loss of generality let $z^{i,j}$ be an optimal contract for which good 1 is a strong medium of exchange. Let good 2 be in zero net-supply, i.e., $k = 2$. If good 2 is a strong medium of exchange for $z^{i,j}$, then the Proposition follows. Therefore consider the case where in which good 2 is not a strong medium of exchange. It will be shown that it is possible to construct an alternative optimal contract $\tilde{z}^{i,j}$ that achieves the same allocation and for which good 2 is a strong medium of exchange. There are two subcases to consider as good 2 can fail to be a strong medium of exchange for $z^{i,j}$ in two ways. Either good 2 already has minimum velocity or there is an alternative optimal contract $\check{z}^{i,j}$ that achieves the same allocation and for which good 2 has minimum velocity. It the follows that either $z_2^{i,j} = 0$ or $\check{z}^{i,j} = 0$ as good 2 is in zero net supply and has minimum velocity.

In what follows we analyze the first subcase as an identical argument established the result for the second subcase. We now define an alternative contract $\tilde{z}^{i,j}$ as follows. First, let $\tilde{z}_1^{i,j}$ be trades with minimum velocity in good one which achieve the original allocation. Such contracts exist by Lemma 1. We now define $\tilde{z}_2^{i,j} = z_1^{i,j} - \tilde{z}_1^{i,j}$. Finally, define $\tilde{z}_m^{i,j} = z_m^{i,j}$ for $m > 2$. Consequently, $z^{i,j}$ and $\tilde{z}^{i,j}$ implement the same allocation. We next verify that $\tilde{z}^{i,j}$ is Bayesian incentive compatible.

Assume by way of contradiction that there exists a state $\bar{\omega}$, and states $\bar{\omega}^j \in E_j(\bar{\omega})$ such that

$$E\left[u\left(e^i(\omega) + \sum_{j\in I} \tilde{z}^{i,j}(\bar{\omega}^j), q(\omega)\right) \;\middle|\; \mathcal{F}_i\right](\bar{\omega}) > E\left[u\left(e^i(\omega) + \sum_{j\in I} \tilde{z}^{i,j}(\omega), q(\omega)\right) \;\middle|\; \mathcal{F}_i\right](\bar{\omega}). \tag{3}$$

Since negative consumption of any good is impossible we must have

$$e^i(\omega) + \sum_{j\in I} \tilde{z}^{i,j}(\bar{\omega}^j) \in \mathbb{R}^n_+. \tag{4}$$

Since good 2 is in zero net-supply $e_2^i(\bar{\omega}) = 0$. Thus, $\sum_{j\in I} \tilde{z}_2^{i,j}(\bar{\omega}^j) \geq 0$ because of (4). Monotonicity of the utility function and (4) therefore imply

$$\begin{aligned}
&E\left[u\left(e^i(\omega) + \sum_{j\in I} z^{i,j}(\bar{\omega}^j), q(\omega)\right) \;\middle|\; \mathcal{F}_i\right](\bar{\omega}) \\
&= E\left[u\left(e_1^i(\omega) + \sum_{j\in I}\left[\tilde{z}_1^{i,j}(\bar{\omega}^j) + \tilde{z}_2^{i,j}(\bar{\omega}^j)\right], 0, \ldots, q(\omega)\right) \;\middle|\; \mathcal{F}_i\right](\bar{\omega}) \\
&= E\left[u\left(e_1^i(\omega) + \sum_{j\in I}\left[\tilde{z}_1^{i,j}(\bar{\omega}^j) + \tilde{z}_2^{i,j}(\bar{\omega}^j)\right], \sum_{j\in I}\tilde{z}_2^{i,j}, \ldots, q(\omega)\right) \;\middle|\; \mathcal{F}_i\right](\bar{\omega}) \\
&\geq E\left[u\left(e_1^i(\omega) + \sum_{j\in I}\tilde{z}_1^{i,j}(\bar{\omega}^j), \sum_{j\in I}\tilde{z}_2^{i,j}, \ldots, q(\omega)\right) \;\middle|\; \mathcal{F}_i\right](\bar{\omega}).
\end{aligned} \tag{5}$$

The first equality follows from the definition of $\tilde{z}^{i,j}$ and from the fact that $z_2^{i,j} = 0$ for all i, j. The second equality follows since agents do not derive

utility from good 2. The inequality holds because of monotonicity.[16] Inequalities (3) and (5), and the fact that $\tilde{z}^{i,j}$ and $z^{i,j}$ implement the same allocation,[17] immediately imply that the $z^{i,j}$ is not Bayesian incentive compatible, a contradiction. Thus, $\tilde{z}^{i,j}$ fulfills Bayesian incentive compatibility.

Finally, it remains to prove that good 2 is a strong medium of exchange for $\tilde{z}^{i,j}$. Assume by way of contradiction that good 2 is not a strong medium of exchange. Then by Definition 6 we can find an alternative optimal contract $\hat{z}^{i,j}$ which implements the same allocation as $\tilde{z}^{i,j}$ and which has minimum velocity in good 2. By construction, $\tilde{z}^{i,j}$ has minimum velocity in good 1. Consequently, Definition 6 implies that $\hat{z}^{i,j}$ has minimum velocity in good 1 as well. This, however, is a contradiction to the fact that good 1 is a strong medium of exchange with respect to the original contract $z^{i,j}$. Consequently, good 2 is a strong medium of exchange for the optimal contract $\tilde{z}^{i,j}$. This concludes the proof.

Proposition 1 shows that a good which is intrinsically worthless can be used as a medium of exchange, provided the good is in zero net-supply. For this result to hold it is crucial that the consumption set is $\mathbb{R}^n_+$. As a consequence the consumption of all goods must be positive. This assumption does not require much justification if a physical good is traded, however, if a good is in zero net-supply the positivity assumption requires a different interpretation. In particular, trading a good which is in zero net-supply means trading IOUs (promises to pay the good) since the physical good itself is not available in positive quantities and therefore cannot be traded directly. The positivity constraint therefore implies that even after misreports, an agent cannot hold net-debt obligations towards other agents.

In the absence of any outside enforcement mechanism (such as auditing), any agent can cover claims against him/herself by writing additional IOUs and thus exiting as a net debtor.[18] Thus, it is necessary to have an auditing

[16] In particular, since $\sum_{j\in I} \tilde{z}_2^{i,j} > 0$, we get $e_1^i(\omega) + \sum_{j\in I}\left[\tilde{z}_1^{i,j}(\bar{\omega}^j) + \tilde{z}_2^{i,j}(\bar{\omega}^j)\right] \geq e_1^i(\omega) + \sum_{j\in I} \tilde{z}_1^{i,j}(\bar{\omega}^j)$. Thus, the assertion follows from monotonicity of u.

[17] Since $z^{i,j}$ and $\tilde{z}^{i,j}$ implement the same allocation, it follows that $E\Big[u\Big(e^i(\omega) + \sum_{j\in I} \tilde{z}^{i,j}(\omega), q(\omega)\Big) \;\Big|\; \mathcal{F}_i\Big](\bar{\omega}) = E\Big[u\Big(e^i(\omega) + \sum_{j\in I} z^{i,j}(\omega), q(\omega)\Big) \;\Big|\; \mathcal{F}_i\Big](\bar{\omega})$.

[18] Obviously, the debt-positions of some other agents cannot clear. As a consequence some agents will be left as creditors with uncovered claims. By that time debtors have already left the economy.

mechanism which verifies whether or not an agent is a net debtor after trade. This, however, requires verifying and recording each agent's trade with *all* other agents. This already requires substantial local auditing. From here it is only a small step to centralized auditing. In particular, all that is required is a meeting of all local auditors where trading records are compared. Such a mechanism completely eliminates the pairwise revelation of information. It corresponds to a mechanism where all agents must first *publicly* agree on the "true" state of nature and trades are then executed accordingly. Such a mechanism does not require a medium of exchange.

We now describe an alternative mechanism which requires relatively little auditing. In particular, each agent i receives $m_i > 0$ units of a common token good with no intrinsic value.[19] The m_i units can be interpreted as a "loan" which must be returned in full after trade. This requires an auditor who collects m_i units of tokens from agent i after trade. It is important to note that the auditor need not have any information or any record of trades. Provided m_i is positive, agents can trade in tokens instead of credit slips. The positivity constraint is automatically enforced by the auditor since each agent i must return at least m_i units of tokens after trade.

It is important to note that even under the above described mechanism the good is a pure medium of exchange in the sense that no direct utility can be derived from consuming the good. As a medium of exchange the good solely fulfills an implicit accounting function. Furthermore, an intrinsic consumption value can interfere with a good's function as a medium of exchange. Thus, whereas it is possible to replace goods by tokens as media of exchange, it is in general not possible to replace tokens by goods without a loss of Pareto efficiency.

5 Concluding Remarks

The paper started with four specific questions. We conclude the paper by discussing these questions in the light of our model and the results. We also indicate some additional questions which the model can address.
(1) The definition of a medium of exchange is based on the insight of Wicksell (1911). However, we require an additional condition which formalizes

[19]The mechanism also works if each agent i receives exactly the same amount of tokens. Thus, the total amount of tokens (provided it is positive) does not have any real effects.

the idea that a medium of exchange is "essential", i.e., the economy cannot function equally without it.
(2) In our model media of exchange arise because of a lack of double coincidence of information rather than just a lack of double coincidence of wants. In particular, Fact 3 in Section 4.2 shows that differential information is crucial.
(3) In our model media of exchange improve the efficiency of the economy. Thus, there are optimal allocations that cannot be achieved without media of exchange. Example 3 illustrates this point.
(4) In our model intrinsically worthless goods can serve as media of exchange under certain conditions. Proposition 1 and the mechanism described thereafter show that a worthless good in the presence of decentralized auditing functions at least as well as any other good as a medium of exchange. This is because these goods serve only an accounting role. It is interesting to note that whereas a token good can always replace a physical good as a medium of exchange, the reverse is not true. In particular, as we point out in Section 4.5, the fact that a good has an intrinsic value can interfere with its function as a medium of exchange. Thus, tokens cannot always be replaced by goods with intrinsic value as a medium of exchange.

This observation is interesting in connection with the often heard assertion that the fact that a government issues and collects taxes in tokens implies that these tokens will be used by the public as a medium of exchange. It is not immediately obvious why this assertion should be true. In particular, agents could pay the government taxes in tokens and use a different good as a medium of exchange. However, as we have pointed out, in our model circulating the tokens reduces the incentive problems in the economy and thus has a Pareto improving effect.

We expect that our model will be useful in addressing additional questions. Gresham's Law is a particularly interesting example. According to Jevons (1875,p.81), Gresham's Law states that bad money drives out good money and that good money cannot drive out bad money. In our model and in Jevons (1875) good money can be interpreted as gold coins of known quality (weight), whereas bad money can be interpreted as gold coins of unknown quality. The validity of Gresham's Law within our model reduces to the question of whether a lottery money is at least as good a medium of exchange as a non-lottery money.

6 Appendix

6.1 Computations for Example 1

Since agent I has complete information, an optimal contract must fulfill two separate individual rationality constraints: one for the good state and the other one for the bad state. In particular, we get

$$5 + z_1^{I,J}(g) + \sqrt{4 + z_2^{I,J}(g) + z_3^{I,J}(g)} \geq 7, \tag{A1}$$

for the good state and

$$5 + z_1^{I,J}(b) + \sqrt{z_3^{I,J}(b)} \geq 5, \tag{A2}$$

in the bad state. In contrast, agent J has no information and consequently assigns the probabilities 0.5 to each state. Thus, the individual rationality constraint of agent J is given by

$$\begin{aligned} 5 + 1/2\Big(-z_1^{I,J}(g) - z_1^{I,J}(b) + \sqrt{2 - z_2^{I,J}(g) - z_3^{I,J}(g)} \\ + \sqrt{2 - z_3^{I,J}(b)}\Big) \geq 5 + \sqrt{2}. \end{aligned} \tag{A3}$$

The incentive constraint for agent I which implies truthful revelation of information in the good state is given by

$$5 + z_1^{I,J}(g) + \sqrt{4 + z_2^{I,J}(g) + z_3^{I,J}(g)} \geq 5 + z_1^{I,J}(b) + \sqrt{4 + z_2^{I,J}(b) + z_3^{I,J}(b)}. \tag{A4}$$

The incentive constraint for the bad state is given by

$$5 + z_1^{I,J}(b) + \sqrt{z_3^{I,J}(b)} \geq 5 + z_1^{I,J}(g) + \sqrt{z_3^{I,J}(g)}. \tag{A5}$$

Further, feasibility requires $-5 \leq z_1^{I,J}(g), z_1^{I,J}(b) \leq 5$, $-4 \leq z_2^{I,J}(g), z_2^{I,J}(b) \leq 0$ and $0 \leq z_3^{I,J}(g), z_3^{I,J}(b) \leq 2$. In order to find the set of all optimal contracts we can therefore solve

$$\max_{z_1^{I,J}, z_2^{I,J}, z_3^{I,J}} -z_1^{I,J}(g) - z_1^{I,J}(b)) + \sqrt{2 - z_2^{I,J}(g) - z_3^{I,J}(g)} + \sqrt{2 - z_3^{I,J}(b)},$$

subject to

$$z_1^{I,J}(g) + \sqrt{4 + z_2^{I,J}(g) + z_3^{I,J}(g)} \geq 2 + a_1, \qquad (A6)$$

$$z_1^{I,J}(b) + \sqrt{z_3^{I,J}(b)} \geq a_2, \qquad (A7)$$

and constraints (A4) and (A5), where $a_1, a_2 \geq 0$ is the surplus which agent I receives in the optimal contract. First, note that we can assume that $z_2^{I,J}(b) = -4$. It is easy to verify that (A4) never binds. Otherwise, inequalities (A3), (A6), and (A7) cannot all hold.[20] Hence, by reducing $z_1^{I,J}(g)$ we can ensure that (A6) binds. Furthermore, by reducing $z_1^{I,J}(b)$ we can ensure that either (A5) or (A7) binds (recall that (A4) never binds). Straightforward computations yield that (A5) must bind.[21] Thus, by inserting (A5) and (A6) in the maximizer we get

$$\max_{z_1,z_2,z_3} \sqrt{2 - z_2^{I,J}(g) - z_3^{I,J}(g)} + \sqrt{2 - z_3^{I,J}(b)} + 2\sqrt{4 + z_2^{I,J}(g) + z_3^{I,J}(g)} + \sqrt{z_3^{I,J}(b)} - \sqrt{z_3^{I,J}(g)}, \qquad (A8)$$

subject to the constraints $-4 \leq z_2^{I,J}(g), z_2^{I,J}(b) \leq 0$, and $0 \leq z_3^{I,J}(g), z_3^{I,J}(b) \leq 2$. First, note that it is optimal to choose $z_3^I(b) = 1$. It is easy to see that no interior solution exists for the remaining parameters, in fact leaving $z_2^{I,J}(g) = z_3^{I,J}(g) = 0$ as the optimal solution.[22] Hence, agents trade goods

[20] Assume (A4) holds with equality. Then insert (A6), (A7) and (A4) into the left-hand side of (A3) and verify that it cannot hold.

[21] Assuming that only (A6) and (A7) bind we can show by inserting (A6) and (A7) in the objective function of the maximization problem and by taking the first order conditions that an optimum requires $z_3^{I,J}(b) = 1$ and $z_2^{I,J}(g) + z_3^{I,J}(g) = -1$. (A7) then implies that $z_1^{I,J}(b) = b - 1$ and (A6) implies $z_1^{I,J}(g) = 2 + a - \sqrt{3}$. We can now insert these values into (A3) to get $2\sqrt{3} - 2 - a - b \geq 2\sqrt{2}$. Since $a, b \geq 0$, this inequality does not hold. Consequently (A5) must bind.

[22] This can be seen easily by considering the derivatives of (A8) with respect to $z_2^{I,J}(g)$ and $z_3^{I,J}(g)$. These derivatives are given by

$$\frac{1}{\sqrt{4 + z_2^{I,J}(g) + z_3^{I,J}(g)}} - \frac{1}{2\sqrt{2 - z_2^{I,J}(g) - z_3^{I,J}(g)}}$$

$$\frac{1}{\sqrt{4 + z_2^{I,J}(g) + z_3^{I,J}(g)}} + \frac{1}{2\sqrt{2 - z_2^{I,J}(g) - z_3^{I,J}(g)}} - \frac{1}{2\sqrt{z_3^{I,J}(g)}}.$$

two and three only in the bad state. The trades in good one are indeterminate and can be used to transfer the available surplus between the two agents. For example, if we choose $z_1^{I,J}(g) = 0$ and $z_1^{I,J}(b) = -1$ then all surplus goes to agent J.

6.2 Proof of Lemma 1

First, define an alternative measure of velocity by

$$\bar{v}^i\left(z_k^{i,j}\right)(\omega) = \sum_{j \in I} \left|z_k^{i,j}(\omega)\right| \Big/ \left(\left|\sum_{j \in I} z_k^{i,j}(\omega)\right| + 1\right). \tag{A9}$$

In order to find trades with minimum velocity with respect to v it is sufficient to prove that the trades have minimum velocity with respect to $\bar{v}$.[23] Note that all trades are $\mathcal{F} = \bigvee_{i,j \in I} \mathcal{F}_i \vee \mathcal{F}_j = \bigvee_{i \in I} \mathcal{F}_i$-measurable. Let A_n, $n \in I\!N$ be the collection of sets forming the partition $\mathcal{F}$. Then every trade $z^{i,j}$ must be constant on each set A_n and can therefore be represented by a sequence $z_n^{i,j}$, $n \in I\!N$. For $i \in I$, let $M_i(\omega) = \bar{v}^i(z_k^{i,j})(\omega)$. Then $M_i(\omega)$ is $\mathcal{F} = \bigvee_{i \in I} \mathcal{F}_i$-measurable. Thus, we can also represent $M_i(\omega)$ by a sequence M_i^n, $n \in I\!N$.

Let $\mathcal{I} = \{(i,j) \mid i \neq j\}$, and let $\mathcal{G} = \{y \mid y{:}\mathcal{I} \times I\!N \to I\!R\}$. Let $F_n = \{y_n{:}\mathcal{I} \to I\!R \mid \sum_{j \in I, j \neq i} y_n(i,j) = \sum_{j \in I, j \neq i} z_k^{i,j}$, for every $i \in I$, and $\bar{v}^i(y_n(i,j)) \leq M_i^n\}$. Then F_n is a compact subset of $I\!R^{\mathcal{I}}$ for every n.[24] Since the weak and the strong topology coincide on the space of sequences, Tychonoff's theorem implies that the set $\{y \in \mathcal{G} \mid y_n \in F_n\}$ is compact. As a consequence, the set $\mathcal{Z} = \{y \in \mathcal{G} \mid y_n \in F_n$ and $y_n(i,j)$ is constant on the partition $\mathcal{F}_i \vee \mathcal{F}_j$ for all $(i,j) \in \mathcal{I}\}$ is compact.

Now define inductively,

$$\mathcal{Y}_{(1,1)} = \arg\min_{y \in \mathcal{Z}} \bar{v}^1(y_1),$$

Consequently, there do not exist values of $z_2^{I,J}(g)$ and $z_3^{I,J}(g)$ for which both derivatives are zero. Furthermore, the derivatives imply that $z_2^{I,J}(g) = z_3^{I,J}(g) = 0$ and $z_2^{I,J}(g) = 4$, $z_3^{I,J}(g) = 2$ are the only candidates for solutions to the maximization problem. However, one can immediately verify that the second candidate solution is not optimal.

[23] In fact, the reverse is not true as the reader is invited to check.

[24] Since $\bar{v}^i$ is continuous, it follows that the set is closed. Furthermore, the set is bounded since $\bar{v}^i((y_n(i,j)) \leq M_i^n$ implies $|y_n(i,j)| \leq M_i^n \left(1 + \sum_{j \in I, j \neq i} z_k^{i,j}\right)$ for all $(i,j) \in \mathcal{I}$.

and in general

$$\mathcal{Y}_{(1,i)} = \arg\min_{y \in \mathcal{Y}_{1,i-1}} \bar{v}^i(y_1)$$

Then $K_1 = \bigcap_{i \in I} \mathcal{Y}_{(1,i)} = \mathcal{Y}_{(1,I)}$ is a compact subset of $\mathcal{G}$. We define K_n inductively by

$$\mathcal{Y}_{(n,1)} = \arg\min_{y \in K_{n-1}} \bar{v}^1(y_n),$$

and in general

$$\mathcal{Y}_{(n,i)} = \arg\min_{y \in \mathcal{Y}_{n,i-1}} \bar{v}^i(y_n),$$

and $K_n = \bigcap_{i \in I} \mathcal{Y}_{(n,i)}$. Since $K_n \supset K_{n+1}$ we get $K = \bigcap_{n \in N} K_n \neq \emptyset$. However, every contract in K is by construction a minimum velocity trade. This concludes the proof.

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